Testing the Strong Equivalence Principle with Mars Ranging Data.

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Abstract

The year 1996 will mark the initiation of a number of new space missions to the planet Mars from which we expect to obtain a rich set of data, including spacecraft radio tracking data. Anticipating these events, we have analyzed the feasibility of testing a violation of the strong equivalence principle (SEP) with Earth-Mars ranging. Using analytic and numerical methods, we have demonstrated that ranging data can provide a useful estimate of the SEP parameter η. Two estimates of the predicted accuracy are quoted, one based on conventional covariance analysis, and the other based on "modified worst case" analysis, which assumes that systematic errors dominate the experiment. If future Mars missions provide ranging measurements with an accuracy of σ meters, after ten years of ranging the expected accuracy for the parameter η will be of order $\sigma_\eta \approx (1-12)\times 10^{-4}\sigma$. In addition, these ranging measurements will provide a significantly improved determination of the mass of the Jupiter system, independent of the test of the SEP polarization effect.

A possible inequality of passive gravitational and inertial masses of the same body leads to an SEP violation, which results in observable perturbations in the motion of celestial bodies. Thus according to the parametrized post-Newtonian (PPN) formalism the ratio of passive gravitational mass m_g to inertial mass m_i of a body with rest mass m, may be written (Nordtvedt, 1968)

$$\frac{m_g}{m_i} = 1 + \eta\frac{\Omega}{mc^2} = 1 - \eta\frac{G}{2mc^2}\int\!\!\int_V d^3z' d^3z'' \frac{\rho(z')\rho(z'')}{|z'-z''|}, \tag{1}$$

where SEP violation is quantified by the parameter η. Note that general relativity, when analyzed in standard PPN gauge (Will, 1993), yields $\eta = 0$. Whereas for the Brans-Dicke theory, for example, $\eta = (2+\omega)^{-1}$, where ω is a free dimensionless parameter of the theory. The quantity Ω is the body's gravitational binding energy. The solar binding energy produces the biggest contribution to the ratio (1) among all the celestial bodies in the solar system. For the standard solar model we obtain $(\Omega/mc^2)_S \approx -3.52\cdot 10^{-6}$, which is almost four orders larger than the Earth's binding energy.

We maintain that a measurement of the solar gravitational to inertial mass ratio can be obtained using Mars ranging data. In order to analyze this effect, we consider the dynamics of the four-body Sun-Mars-Earth-Jupiter (or S-M-E-J) system in the solar system barycentric inertial frame. The quasi-Newtonian acceleration of Mars with respect to the Sun, $\ddot{\vec{r}}_{SM}$, is straightforwardly calculated to be

$$\ddot{\vec{r}}_{SM} = \ddot{\vec{r}}_M - \ddot{\vec{r}}_S = -\mu^*_{SM}\cdot\frac{\vec{r}_{SM}}{r^3_{SM}} + \mu_J\Big[\frac{\vec{r}_{JS}}{r^3_{JS}} - \frac{\vec{r}_{JM}}{r^3_{JM}}\Big] + \eta\Big(\frac{\Omega}{mc^2}\Big)_S\mu_J\frac{\vec{r}_{JS}}{r^3_{JS}}, \tag{2}$$

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where $\mu^*_{SM} \equiv \mu_S + \mu_M + \eta\Big[\mu_S\Big(\frac{\Omega}{mc^2}\Big)_M + \mu_E\Big(\frac{\Omega}{mc^2}\Big)_S\Big]$ and $\mu_k \equiv Gm_k$. The first and second terms on the right side of (2) are the classical Newtonian and tidal acceleration terms respectively. We denote the last term in this equation as $\vec{a}_\eta$. This is the SEP acceleration term of order c^{-2} which may be treated as a perturbation on the restricted three-body problem.[4] The corresponding SEP effect is evaluated as an alteration of the planetary Keplerian orbit. To good approximation the SEP acceleration $\vec{a}_\eta$ has constant magnitude and points in the direction from Jupiter to the Sun, and because it depends only on the mass distribution in the Sun, both Earth and Mars experience the same perturbing acceleration. The orbital responses of each of these planets to the term $\vec{a}_\eta$ determines the perturbation in the Earth-Mars distance and allows a detection of the SEP parameter η by means of ranging data.

The presence of the acceleration term $\vec{a}_\eta$ in the equations of motion (2) results in a polarization of the orbits of Earth and Mars, exemplifying the planetary SEP effect. By analyzing the effect of a non-zero η on the dynamics of the Earth-Moon system moving in the gravitational field of the Sun, Nordtvedt (1968) derived a polarization of the lunar orbit in the direction of the Sun with amplitude $\delta r \sim 13\,\eta$ meters (Nordtvedt effect). The most accurate test of this effect is presently provided by Lunar Laser Ranging (LLR), and in recent results, Dickey *et al.* (1994) obtain

$$\eta = -0.0005 \pm 0.0011\,. \tag{3}$$

Results are also available from numerical experiments with combined processing of LLR, spacecraft tracking, planetary radar and Very Long Baseline Interferometer (VLBI) data (Chandler *et al.*, 1994). Note that the Sun-Mars-Earth-Jupiter system, though governed basically by the same equations of motion as the Sun-Earth-Moon system, is significantly different physically. For a given value of SEP parameter η the polarization effects on the Earth and Mars orbits are almost two orders of magnitude larger than on the lunar orbit. We have examined the SEP effect on reduced Earth-Mars ranging generated by the Deep Space Network (DSN) during the *Mariner 9* and *Viking* missions. Moreover, future Mars missions, now being planned as joint U.S.-Russian endeavours, should yield additional ranging data.

Using both analytic and numerical methods, we have determined the accuracy with which the parameter η can be measured using Earth-Mars ranging. The following set of the parameters were included in a covariance analysis; $r_{E_0}, r_{M_0}, p_{Er_0}, p_{Mr_0}, p_{E\theta_0}, p_{M\theta_0}$, $\theta_{E_0} - \theta_{M_0}, \mu_S, \mu_J$ and $|\vec{a}_\eta|$, where r_{B_0} and p_{Br_0} are the initial barycentric distance and corresponding momentum of planet B. Note that Jupiter's mass μ_J was taken as an unknown. This is because the octopolar tide of Jupiter acting on the orbits of Earth and Mars produces polarizations similar to those produced by the SEP effect, but fortunately with a different contribution to both orbits and therefore separable from the desired SEP effect. If Jupiter's mass were uncertain by 4 parts in 10^8, its tidal polarization of Mars' orbit would be uncertain by an amount equivalent to $\eta \sim 0.001$, for example. But Jupiter's mass is only known to about one part in a million, so one must include μ_J as a free parameter in analysis of the Mars ranging experiment. Although we performed both analytical and numerical error analyses, we obtained the most reliable results using a numerical integration of (2) along with its counterpart for Earth.

While numerical integration is expected to be more accurate than analytic approximations, it is possible to gain some insight into the planetary SEP effect by working to first order in the eccentricity and by doing a realistic analytical calculation using elliptical reference orbits for Earth and Mars. But we found that by using the method of variation of parameters, we could calculate the perturbed orbits of Earth and Mars to fourth order in the eccentricity. This revealed that the eccentricity correction

[4]While it is not the only term of that order, the other post-Newtonian c^{-2} terms (suppressed in equation (2)) do not affect the determination of η until the second post-Newtonian order ($\sim c^{-4}$).

plays a more significant role than one might expect. One reason for this is that the eccentricity corrections include additional secular terms proportional to the time. Such elements dominate at large times, and the eccentricity corrections thereby qualitatively change the nature of the solution in the linear approximation.

In order to obtain results for σ_η, we assumed that N daily range measurements are available from a Mars mission, each measurement having the same uncertainty σ in units of meters. The initial angles between Earth and Jupiter and Mars and Jupiter were taken from the JPL emphemeris (DE242) at time 2441272.75, the beginning of the *Mariner 9* ranging measurements. We found that the uncertainty in η first drops very rapidly with time and then after a few years approaches an asymptotic behavior $\sim N^{-1/2}$. This behavior gives a lower bound on the uncertainty as predicted by conventional covariance analysis. For a mission duration of order ten years, the uncertainty behaves as

$$\sigma_\eta \sim 0.0039\sigma/\sqrt{N}. \tag{4}$$

This result is valid for Gaussian random ranging errors with a white spectral frequency distribution. However, past ranging measurements using the *Viking Lander* have been dominated by systematic error (Chandler *et al.*, 1994). One approach to accounting for systematic error is to multiply the formal errors from the covariance matrix by $\sqrt{N}$ (Nordtvedt, 1978). With this approach, the expected error decreases rapidly near the beginning of the data interval, but for large N approaches an asymptotic value. However, we believe this is overly conservative. A more optimistic error estimate would include a realistic description of the time history of the systematic error. But a realistic systematic error budget for ranging data to Mars, or for Mercury as considered by a group at the University of Colorado (P. Bender, private communication), is not presently available. Yet it is unlikely that we will be so unfortunate that the frequency spectrum of the signal will match the spectrum of the systematic error. Hence we reduce the upper error bound determined by the $\sqrt{N}$ multiplier ($\sigma_\eta = 0.0039\sigma$) by a numerical factor. The ranging experiment proposed by the Colorado group for Mercury is quite similar to our proposed experiment using Mars. We therefore follow the Colorado group and reduce the worst-case error estimate by a factor of three and call the result the modified worst-case analysis. This yields an asymptotic value for the error given by $\sigma_\eta = 0.0013\sigma$, in our opinion a realistic estimate of the upper error bound.

The covariance analysis gives the expected formal error in μ_J as well. For a mission time of order ten years we find $\sigma_{\mu_J} \sim 5.7\,\sigma/\sqrt{N}$ in km^3s^{-2}, where N, as before, is the number of daily ranging measurements taken during the mission. For $\sigma = 7.9$ m, σ_{μ_J} falls below the present accuracy determined from the *Pioneer 10* and *11* and *Voyager 1* and *2* flybys, namely $\mu_J = 100\ km^3s^{-2}$ (Campbell & Synnott, 1985), after two years of Mars ranging. Earth-Mars ranging can provide an impoved determination for the mass of the Jupiter system, independent of any determination of the SEP effect.

For existing Mars ranging derived from the *Mariner 9, Viking*, and *Phobos* missions, the *rms* ranging residual referenced to the best-fit Martian orbit is 7.9 m. We have computed the covariance matrix with assumed daily ranging measurements for *Mariner 9* (actual data interval JD 2441272.750 to JD 2441602.504) and *Viking* (actual data interval JD 2442980.833 to JD 2445286.574). Additionally, one ranging measurement from *Phobos* (actual time JD 2447605.500) was included, although it had negligible effect on the result. With $\sigma = 7.9$ m, a formal error $\sigma_\eta = 0.0012$ is obtained from the covariance matrix. With 7.2 years of Mars ranging, even though not continuous, the asymptotic limit of the modified worst-case analysis implies a realistic error $\sigma_\eta = 0.02$, which is about 17 times the formal error. We conclude that the best determination of η is provided by the LLR data, but the existing Mars ranging data provide an independent solar test within a realistic accuracy interval of

$$\sigma_\eta \approx 0.0012 - 0.02 \qquad (Mariner\ \ 9,\ \ Viking,\ \ Phobos) \tag{5}$$

Future Mars Orbiter and Lander missions are expected to achieve an *rms* systematic ranging error between 0.5 and 1.0 m. This implies modified worst-case realistic errors for η and μ_J of $\sigma_\eta \sim 0.0012\sigma$ and $\sigma_{\mu_J} \sim 1.9\,\sigma\ km^3s^{-2}$. Hence with mission durations of order ten years, the interval for the uncertainties σ_η and σ_{μ_J} should be

$$\sigma_\eta \approx (0.0001 - 0.0012)\ \sigma,$$
$$\sigma_{\mu_J} \approx (0.09 - 1.9)\ \sigma \quad \mathrm{km}^3\mathrm{s}^{-2}, \tag{6}$$

where the lower bound is based on random errors and conventional covariance analysis, while the upper bound represents the modified worst-case results as described in the paragraph following (4). The expected accuracy of future ranging experiments should put significant constraints on theoretical models, including a possible inequality of the solar inertial and gravitational masses. Although we have shown that a determination of η with existing *Mariner 9* and *Viking* ranging data is of some interest, the expected accuracy given by (5) does not motivate us to place a high priority on a lengthy and difficult reanalysis of those data. Instead, we are planning on participating in future Mars missions with the goal of generating ranging data as free of systematic error as possible, and extending over as many years as possible. Along the way, we most likely will obtain a test of the SEP with existing Mars ranging. We believe William Fairbank would have encouraged us to pursue such an analysis, if only because we might just be surprised by the result.

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